Optimal Structural Modifications to Enhance the Active Vibration Control of Flexible Structures

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This study provides a method of vibration control of large space structures by simultaneously integrating the structure and control design to reduce the structural response from a disturbance. The formulation of the design scheme is obtained by the structural modification of some nominal finite element model, which is controlled in an optimal fashion by a linear regulator, to increase the active modal damping factor beyond that of the nominal structure. The structural modifications are achieved by using a nonlinear mathematical optimization technique. The objective function is the weight of the structure with a constraint on the damping parameter of the closed-loop system. The application of the algorithm is illustrated by designing an ACOSS-FOUR model with different constraint values.

Introduction

LARGE space structures face difficult problems of vibration control. Because of the requirement for low weight, such structures will lack the stiffness and damping necessary for the passive control of vibrations. Therefore, a great deal of research is currently in progress on designing active vibration control systems for such structures.[1] The objective of vibration control is to design the structure and its controls to reduce the mean square response of the structure to a desired level within a reasonable span of time. In addition, it is important that this objective be achieved in some optimal manner. For a structural designer, the optimal design represents an adjustment of the structural parameters to minimize the structural mass while improving the dynamic characteristics to reduce the dynamic response from some initiating disturbance. For a control designer, optimization represents sizing and placement of actuators and sensors in a manner that a specified performance index is minimized.

The dynamic response of a structure depends upon the global stiffness of the structure and the active control system designed for that structure. The global stiffness of the structure is a function of the material used in the fabrication, the geometry of the structure and the cross-sectional areas of the members. With the material and geometry fixed, the variables available to the designer to improve the dynamic performance are the cross-sectional areas of the members and the optimum active control system. The cross-sectional areas of the members determine the weight as well as the stiffness of the structure. In the conventional approach, structural design and structural control system design are essentially uncoupled as the interaction between the structures and controls designers has been very minimal in the past. The structural designer establishes a nominal structural design based upon strength and stiffness requirements obtained from anticipated peak maneuver loads expected during the operation of the space structure. His primary concern is to design a lightweight structure that will satisfy the strength, stiffness, and other performance requirements. In general, the designer of the active control system has little input in the evolution of the basic structural design. The control system design is undertaken for a specified nominal structure where the structural analyst's participation in the control design is limited to providing information about the frequencies and mode shapes of the nominal structure. If difficulty in achieving a satisfactory controlled structure results, there does not presently exist systematic methods for modifying the nominal structure to enhance the performance of the controlled structure.

The optimum design of large space structures (LSS) was recently investigated using two approaches. In the first,[2-4] an optimum structure is initially designed to satisfy constraints on weight, strength, displacements, frequency distribution, etc., and then an optimum control system is designed to improve the dynamic response of the structure. In the second approach,[5-7] a simultaneous integrated design of the structure and vibration control system is achieved by improving the configuration as well as the control system. However, the applications in Refs. 5-7 are for simple structures with a small number of design variables. A unified approach to achieve satisfaction of eigenspace constraints is presented in Refs. 8 and 9.

This paper discusses application of the control methods directly during the structural design to achieve better response through passive structural stiffness modifications. A multivariable control theory is applied to a nominal structural design to determine the optimal desired control system characteristics for achieving a desired response. The particular controls approach used is linear quadratic regulator theory with constant gain feedback. The modified structure is obtained by changing the cross-sectional areas of the members so that the weight of the structure is minimum and the modal damping factors associated with the real part of the closed-loop eigenvalues of the optimally controlled structure have specified values. The optimization is carried out by using a nonlinear mathematical programming technique.

Two structural models were selected for this study. The first is a two-bar truss (Fig. 1) contrived for its simplicity and small number of design variables. The second is a tetra-

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hedral truss (Fig. 2) devised by the Charles Stark Draper Laboratory[10] as one of the simplest nonplanar geometries capable of representing a large space structure. This structure, ACOSS-FOUR, in spite of its simplicity, models the feed tower in a generic class of large-antenna applications. The apex of the structure (node 1) represents the antenna feed and its motion in the X-Y plane has to be actively controlled to improve the performance of the structure line of sight (LOS). A previous study[2,4] by the authors was concerned with the effects of structural modifications on the dynamic behavior of this representative space structure with and without active controls. Modifications to the structure were made in order to minimize the weight with constraints on the frequency distribution, but without any simultaneous integrated design of the structure and the control system.

Equations of Motion

The equations of motion for a large space structure with no external disturbance are given by

$$[M]\{\ddot{U}\}+[E]\{\dot{U}\}+[K]\{U\}=[D][F] \tag{1}$$

where $[M]$ is the mass matrix, $[E]$ the damping matrix, and $[K]$ the total stiffness matrix. The matrices are $n\times n$, where n is the number of degrees of freedom of the structure. In Eq. (1), $[D]$ is the $n\times p$ applied load distribution matrix relating the control input vector $\{F\}$ to the coordinate system. The number of elements in $\{F\}$ are assumed to be p. $\{U\}$ in Eq. (1) is a vector defining the amplitude of motion. Introducing the coordinate transformation,

$$\{U\}=[\phi]\{\eta\} \tag{2}$$

where $[\phi]$ is the $n\times n$ model matrix whose columns are the eigenvectors and $\{\eta\}$ the modal coordinate system, Eq. (1) can be transformed into n uncoupled differential equation as

$$[\bar{M}]\{\ddot{\eta}\}+[\bar{E}]\{\eta\}+[\bar{K}]\{\eta\}=[\phi]^T[D][F] \tag{3}$$

where

$$[\bar{M}]=\ulcorner I \lrcorner \tag{4}$$

$$[\bar{E}]=\ulcorner 2\zeta\omega \lrcorner \tag{5}$$

$$[\bar{K}]=\ulcorner \omega^2 \lrcorner \tag{6}$$

In Eq. (5), $\{\zeta\}$ is the vector of the modal damping factors and ω the structural circular frequency. Equation (3) can be converted into a state space representation by using the transformation

$$\{x\}_{2n}=\left\{\frac{\eta}{\dot{\eta}}\right\}_{2n} \tag{7}$$

where $\{x\}$ is the state variable vector. This gives

$$\{\dot{x}\}=[A]\{x\}+[B]\{f\} \tag{8}$$

where $[A]$ is a $2n\times 2n$ plant matrix, $[B]$ a $2n\times p$ input matrix, and $\{f\}$ the $p\times 1$ control vector. The plant matrix and the input matrix are given by

$$[A]=\left[\begin{array}{c|c} 0 & I \\ \hline -\omega^2 & -2\zeta\omega \end{array}\right] \tag{9}$$

$$[B]=\left[\frac{0}{\phi^T D}\right] \tag{10}$$

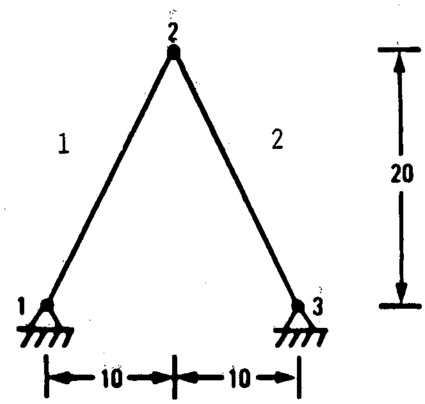

Fig. 1 Two-bar truss.

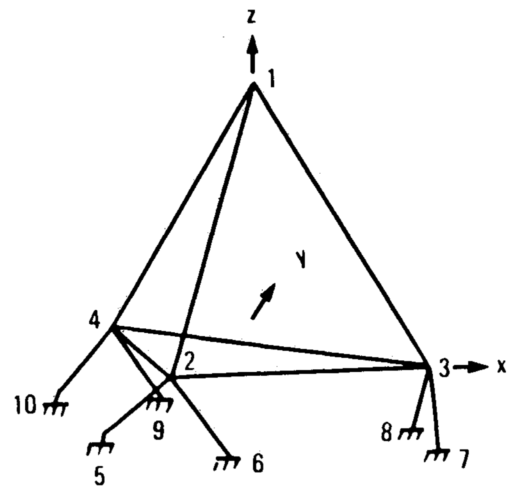

Fig. 2 ACOSS-FOUR finite element model.

Equation (8) is the state input equation. The state output equation is given by

$$\{y\}=[C]\{x\} \tag{11}$$

where $\{y\}$ is a $q\times 1$ output vector and $[C]$ a $q\times 2n$ output matrix. If the sensors and the actuators are colocated, then $q=p$ and

$$[C]=[B]^T \tag{12}$$

In order to design a controller using a linear quadratic regulator, a performance index (PI) can be defined as[11]

$$\mathrm{PI}=\int_0^t(\{x\}^T[Q]\{x\}+\{f\}^T[R]\{f\})\mathrm{d}t \tag{13}$$

where $[Q]$ is the state weighting matrix, which has to be positive semidefinite, and $[R]$ the control weighting matrix, which has to be positive definite. By proper selection of the elements of these matrices, it is possible to control the damping response time and amplitudes of vibrations, etc., of the system. The use of PI in Eq. (13) represents a compromise between the minimum error and the minimum energy criteria. The result of minimizing the quadratic performance index and satisfying the state equation gives the state feedback control law

$$\{f\}=-[\tilde{G}]\{x\} \tag{14}$$

where $[\tilde{G}]$ is the optimum gain matrix given by

$$[\tilde{G}]=[R]^{-1}[B]^T[\tilde{P}] \tag{15}$$

$[\tilde{P}]$ is a symmetric positive definite matrix called the Riccati matrix and is obtained by the solution of the algebraic Riccati equation,[11]

$$[A]^T[P]-[P][B][R]^{-1}[B]^T[P]+[P][A]+[Q]=0 \tag{16}$$

Substituting Eq. (14) in Eq. (8) gives the governing equations for the optimal closed-loop system in the form

$$\{\dot{x}\}=[A]_{c\ell}\{x\} \tag{17}$$

where

$$[A]_{c\ell}=[A]-[B][\bar{G}] \tag{18}$$

The solution of Eq. (18) can be obtained by using available subroutines. The eigenvalues of the closed-loop matrix $[A]_{c\ell}$ are a set of complex conjugate pairs written as

$$\lambda_i=\tilde{\sigma}_i \pm j\tilde{\omega}_i \quad i=1,...,2n \tag{19}$$

The damping factors ξ_i and the damped frequencies $\tilde{\omega}_i$ are related to the complex eigenvalues through

$$\xi_i=-\tilde{\sigma}_i/(\tilde{\sigma}_i^2+\tilde{\omega}_i^2)^{1/2} \tag{20}$$

The effective damping response time is given by

$$T_e=\frac{J}{E_0}=\frac{\{x_0\}^T[\tilde{P}]\{x_0\}}{\{x_0\}^T[Q]\{x_0\}} \tag{21}$$

where $\{x_0\}$ is the initial value of the state variable vector at $t=0$. The magnitude of T_e indicates the effect of the control system on reducing vibrations. If T_e is large, then the initial energy would be dissipated slowly, thus yielding a longer damping response time. A small value of T_e would indicate that the initial energy would be dissipated quickly, thus yielding a shorter damping response time.

Optimization Procedure

The selection of the cross-sectional areas of the members of the optimum design was accomplished by solving the following minimization problem. Minimize weight,

$$W=\Sigma\rho_i A_i \ell_i \tag{22}$$

such that

$$g_j(\xi_i)=0 \tag{23}$$

$$g_j(A_i)\geq 0 \tag{24}$$

where $g_j(\xi_i)$ represents equality constraints on the closed-loop damping ratio in the form

$$g_j(\xi_i)=\xi_i-\bar{\xi}_i=0 \tag{25}$$

and a lower limit on the cross-sectional areas of the members A_i in the form

$$g_j(A_i)=A_i-A_i(\min)\geq 0 \tag{26}$$

In Eq. (22), ρ_i is the density of the material and ℓ_i the volume parameter associated with the ith element. In Eq. (25), ξ_i is the damping factor of the closed-loop system defined in Eq. (20) and $\bar{\xi}_i$ is the desired value of the damping factor for the ith frequency. The optimization problem was solved by using the VMCON[12] optimization subroutine based upon Powell's algorithm for nonlinear constraints that uses Lagrangian functions.[13] This subroutine requires the gradients of the objective function and the constraints. The gradients of the objective function W can be explicitly written by differentiating Eq. (22) with respect to the design variable A_i. The gradients of the modal damping parameter ξ_i for the closed-loop system were determined by using a finite difference technique.

The major steps involved in obtaining an optimum design were as follows:

1) Assign initial sizes to all the elements.

2) Assemble the matrices $[K]$ and $[M]$ and determine the structural eigenvalues and eigenvectors.

3) Use the eigenvalues and eigenvectors to assemble the plant matrix $[A]$ and input matrix $[B]$ in Eq. (8).

4) Solve the linear optimum control problem and determine the eigenvalues of the closed-loop matrix $[A]_{c\ell}$ in Eq. (18).

5) Determine the gradients of the damping factors ξ_i defined in Eq. (20) by perturbing each design variable A_i one at a time. In the present case, each area was increased by 5%.

6) Use the VMCON subroutine to determine the modified values of the design variables A_i.

7) Terminate the iterative procedure after a specified number of iterations or when the weight difference between consecutive iterations is small. Otherwise go to step 2.

Illustrative Examples

The algorithm discussed above was applied to two structures. The elements of the structures were represented by bar elements that allow only axial deformation. The dimensions of the structures were defined in unspecified consistent units. The elastic modulus of the members was assumed to be 1.0 and the density of the structural material to be 0.001. The weighting matrices $[Q]$ and $[R]$ in the definition of the performance index [Eq. (13)] were each assumed to be equal to the identity matrix. In practice, more specific values of $[Q]$ and $[R]$ are generally selected, primarily based upon intuition; but, here the selection as the identity matrices was made since the purpose of the present investigation was to improve vibration control through structural modifications. Other values of $[Q]$ and $[R]$ could be made and, in that case, the necessary structural modifications would then be dependent upon that selection of the weighting matrices.

Example 1: Two-Bar Truss

The two-bar truss shown in Fig. 1 was selected for its simplicity. A nonstructural mass of two units was attached at node 2. The actuator and the sensor were located in element 1 connecting nodes 1 and 2. The cross-sectional areas of members 1 and 2 of the initial design (design A) were assumed to be 1000 and 100, respectively. The minimum size constraint was set equal to 10 for both members. Design B was obtained by minimizing the weight of the structure with the constraint that the damping parameter ξ_1 associated with the lowest frequency of the closed-loop system must be the same as that of the initial design (design A). The iteration history for this case is given in Table 1. The initial weight of the structure was reduced from 24.60 to 14.92 after 11 iterations. Additional iterations did not reduce the weight any further. The damping parameter ξ_1 of the closed-loop system was nearly the same for all iterations. The damping parameter ξ_1 associated with the second frequency increased from 0.07483 to 0.09497. The structural frequencies ω_1 and ω_2 for all the iterations are also given in Table 1. For design C, a constraint on ξ_1 was imposed to increase its value by 10% (i.e., $\xi_1=0.02138$) over that of the initial design A. The iteration history for this case is shown in Table 2. For design C, the weight of the structure increased from that of design A. The final design weighed 25.02 and the optimum design was obtained in only five iterations. The damping parameter ξ_2 associated with the second frequency did not change substan-

Table 1 Iteration history for two-bar truss (design B)

Iteration	Weight	A_1	A_2	ξ_1	ξ_2	ω_1	ω_2
1	24.60	1000.0	100.0	0.01944	0.07483	1.17	4.82
2	24.59	999.97	99.99	0.01944	0.07483	1.17	4.82
3	24.59	999.84	99.95	0.01944	0.07483	1.17	4.82
4	24.57	999.17	99.75	0.01944	0.07483	1.17	4.82
5	24.47	995.84	98.76	0.01944	0.07500	1.17	4.81
6	23.99	979.23	93.86	0.01944	0.07572	1.14	4.76
7	21.76	902.09	71.22	0.01944	0.07933	0.995	4.56
8	19.91	836.98	53.36	0.01946	0.08278	0.864	4.38
9	17.46	750.15	31.10	0.01943	0.08809	0.662	4.13
10	14.94	658.11	10.10	0.01941	0.09491	0.377	3.85
11	14.92	657.72	10.00	0.01944	0.09447	0.377	3.84

Table 2 Iteration history for two-bar truss (design C)

Iteration	Weight	A_1	A_2	ξ_1	ξ_2	ω_1	ω_2
1	24.60	1000.0	100.0	0.01944	0.07483	1.17	4.82
2	25.02	991.90	127.31	0.02137	0.07464	1.32	4.83
3	25.03	991.82	127.45	0.02138	0.07464	1.32	4.83
4	25.02	991.79	127.44	0.02138	0.07464	1.32	4.83
5	25.02	991.64	127.38	0.02138	0.07465	1.32	4.83

Table 3 Closed-loop eigenvalues for two-bar truss

Design A		Design B		Design C	
Real part	Imaginary part	Real part	Imaginary part	Real part	Imaginary part
−0.3610	±4.81	−0.365	±3.83	−0.360	±4.81
−0.0228	±1.17	−0.0073	±0.377	−0.0282	±1.32

tially from the initial design. The fundamental frequency ω_1 increased from 1.17 to 1.32. The closed-loop eigenvalues for the three designs are given in Table 3.

Example 2: ACOSS-FOUR

The finite element model of ACOSS-FOUR is shown in Fig. 2. The edges of this tetrahedral truss are 10 units long. The structure has 12 degrees of freedom and 4 masses of 2 units each are attached at nodes 1-4. The coordinates of the node points are given in Table 4. The actuators and the sensors are located in six bipods, which are assumed to coincide with each other. Thus, matrix $[D]$ in Eq. (3) would consist of the direction cosines relating the force in the six bipods with their components in the coordinate directions. Since the sensors and the actuators are colocated, matrices $[B]$ and $[C]$ in Eqs. (8) and (11) will satisfy Eq. (12). The weighting matrix $[R]$ for this case would be a 6×6. The passive damping parameter $\{\zeta\}$ in Eq. (9) was assumed to be zero.

The nominal design is denoted by design A with cross-sectional areas of the members equal to those assigned by the Charles Stark Draper Laboratory[10] for their preliminary investigation. This design was used as the initial design in the optimization program. The cross-sectional areas of the members for this design are given in Table 5. The squares of the structural frequencies for all 12 modes for design A are given in Table 6. This design weighed 43.69.

Design B was obtained with a constraint on the damping parameter ξ_1 of the closed-loop system associated with the lowest frequency. It was specified that ξ_1 for the optimized design must be equal to ξ_1 of design A. For design A this value is equal to 0.05464. The iteration history for this case is shown in Table 7. This table also shows the change in the real and imaginary part of λ_1 and also ξ_1 with each iteration. The initial weight of the structure was 43.70 and the optimum design weighed 39.58. It took about 17 iterations to arrive at this design and the constraint was satisfied as an equality constraint for most of the iterations. The cross-sectional areas of the members and the structural frequencies for Design B are given in Tables 5 and 6 respectively.

Design C was obtained with the constraint that the damping parameter ξ_1 of the closed-loop system be increased to 0.3 from 0.05464 of the initial design A. The iteration history for this case is given in Table 8. The weight of the optimized structure was 47.46, which was higher than the initial design. The final design was obtained after 11 iterations (Table 8). Table 8 also shows the changes in the real and imaginary parts of the closed-loop eigenvalues and the modal damping parameter. After the sixth iteration, the modal damping remained constant. The cross-sectional areas and the structural frequencies for design C are given in Tables 3 and 6, respectively.

Table 4 Node point coordinates for ACOSS-FOUR

Node	X	Y	Z
1	0.0	0.0	10.165
2	−5.0	−2.887	2.00
3	5.0	−2.887	2.00
4	0.0	5.7735	2.00
5	−6.0	−1.1547	0.0
6	−4.0	−4.6188	0.0
7	4.0	−4.6188	0.0
8	6.0	−1.1547	0.0
9	−2.0	5.7735	0.0
10	2.0	5.7735	0.0

Table 5 Cross-sectional areas of members of ACOSS-FOUR

Element	Design A	Design B	Design C
1(1-2)	1000.	936.10	998.22
2(2-3)	1000.	936.10	998.62
3(1-3)	100.	40.07	354.69
4(1-4)	100.	40.07	354.79
5(2-4)	1000.	936.10	998.62
6(3-4)	1000.	936.10	998.22
7(2-5)	100.	79.15	14.74
8(2-6)	100.	79.16	14.73
9(3-7)	100.	75.10	15.34
10(3-8)	100.	81.20	46.65
11(4-9)	100.	75.10	15.34
12(4-10)	100.	81.20	46.74
Weight	43.69	39.58	47.56

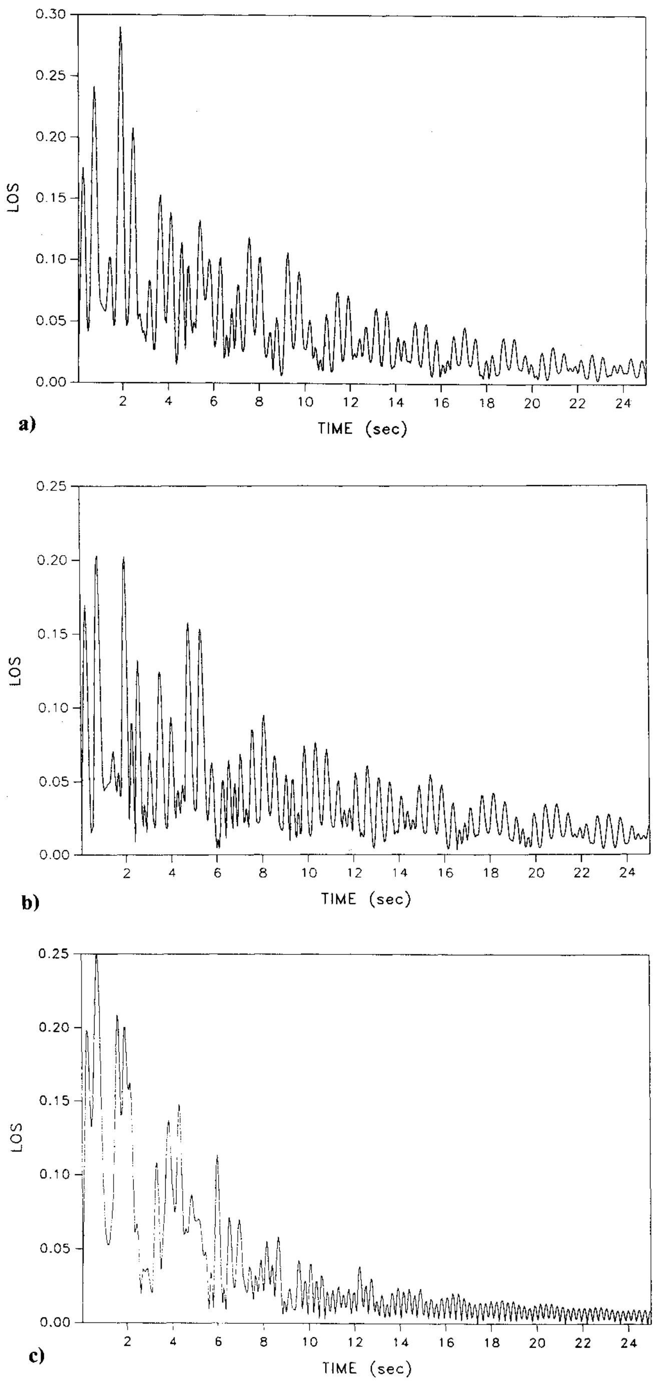

Fig. 3 LOS transient response of ACOSS-FOUR: a) design A, b) design B, c) design C.

The open- and closed-loop eigenvalues for the three designs are given in Tables 9 and 10, respectively. Table 11 contains the modal damping parameters associated with the closed-loop system for all the modes of the three designs.

Comparing the three designs, the following observations can be made:

1) Optimization of the tetrahedral truss with a constraint on the modal damping parameter of the closed-loop system has reduced the fundamental structural frequencies. (See Table 6.) For design A, ω_1 was equal to 1.801, which was reduced to 0.8462 for design B and 0.7283 for design C.

2) For design A, the structural frequencies associated with modes 3 and 4 and modes 7 and 8 are close to each other. However, for design B, only the frequencies associated with modes 7 and 8 are close to each other. For design C, the frequencies are well spread out and no two frequency values are as close as those for designs A and B. (See Table 6.)

Table 6 Natural frequencies ω_i^2 of ACOSS-FOUR

Mode	Design A	Design B	Design C
1	1.801	0.8462	0.7283
2	2.771	1.514	1.074
3	8.356	5.719	1.734
4	8.746	6.700	2.147
5	11.55	9.136	3.754
6	17.68	13.75	6.101
7	21.73	15.40	25.04
8	22.61	15.54	34.76
9	72.92	66.04	68.91
10	85.57	78.14	81.05
11	105.8	98.07	101.0
12	166.5	154.8	171.2

Table 7 Iteration history for ACOSS-FOUR (design B)

Iteration	Weight	λ_1 Real	λ_1 Imaginary	ξ_1
1	43.70	−0.0734	±1.34	0.05464
2	43.69	−0.0734	±1.34	0.05464
3	43.69	−0.0734	±1.34	0.05464
4	43.67	−0.0733	±1.34	0.05464
5	43.60	−0.0730	±1.33	0.05464
6	43.19	−0.0712	±1.30	0.05464
7	41.60	−0.0628	±1.15	0.05431
8	41.67	−0.0637	±1.16	0.05467
9	40.42	−0.0552	±1.03	0.05434
10	40.42	−0.0562	±1.03	0.05464
11	40.42	−0.0562	±1.03	0.05464
12	40.41	−0.0560	±1.03	0.05464
13	40.39	−0.0560	±1.02	0.05464
14	40.27	−0.0552	±1.01	0.05464
15	39.65	−0.0507	±0.929	0.05464
16	39.64	−0.0508	±0.928	0.05464
17	39.58	−0.0503	±0.920	0.05464

Table 8 Iteration history for ACOSS-FOUR (design C)

Iteration	Weight	λ_1 Real	λ_1 Imaginary	ξ_1
1	43.70	−0.0734	±1.34	0.0546
2	43.74	−0.3060	±0.722	0.3899
3	47.60	−0.2740	±0.840	0.3105
4	47.57	−0.2700	±0.861	0.2922
5	47.57	−0.2700	±0.859	0.3001
6	47.57	−0.2700	±0.859	0.3000
7	47.56	−0.2700	±1.859	0.3000
8	47.52	−0.2700	±0.859	0.3000
9	47.56	−0.2700	±0.859	0.3000
10	47.54	−0.2700	±0.859	0.3000
11	47.46	−0.2700	±0.859	0.3000

3) The modal damping parameters associated with most of the modes have increased in designs B and C from those of design A. The percentage increase is higher for the lower modes than it is for the higher modes. (See Table 11.)

In order to compare the dynamic behavior, the three designs are subjected to the same initial condition. A unit displacement was imposed at node 2 in the X direction at $t=0$. The transient response was simulated by finding the solution to Eq. (17) for the period $t=0$-25 s at time intervals of $t=0.05$ s. The magnitude of LOS was calculated at each time interval. LOS is equal to the movement of the apex (node point 1) from the Z axis. It is equal to $[(\mathrm{LOS}-X)^2+(\mathrm{LOS}-Y)^2]^{1/2}$, where LOS − X and LOS − Y are the components of LOS in the X and Y coordinate directions. The time histories for the three designs are given in Fig. 3. Figure 4 contains a plot of the performance index vs time. The total performance index which is equal to the area under the curve for the three designs is given in Table 12.

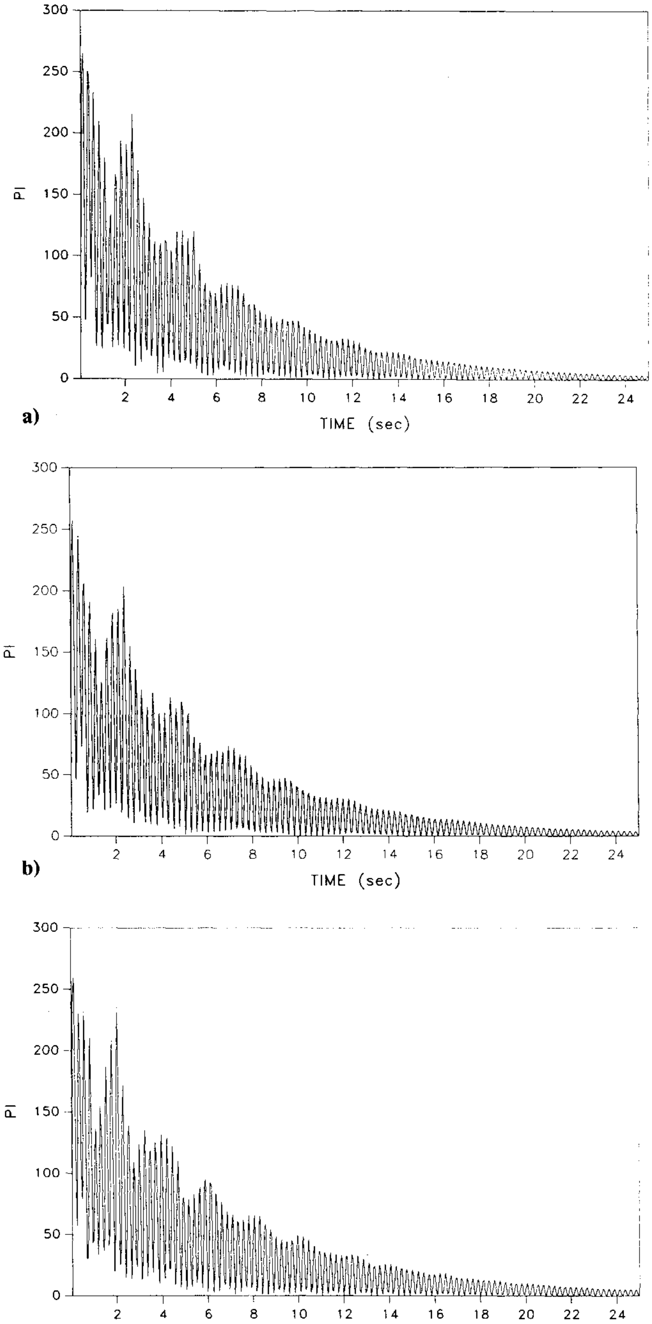

Fig. 4 Variation of the performance index for ACOSS-FOUR: a) design A, b) design B, c) design C.

Table 9 Open-loop eigenvalues for ACOSS-FOUR

Design A		Design B		Design C	
Real part	Imaginary part	Real part	Imaginary part	Real part	Imaginary part
0.0	± 1.34	0.0	± 0.92	0.0	± 0.853
0.0	± 1.66	0.0	± 1.23	0.0	± 1.04
0.0	± 2.89	0.0	± 2.39	0.0	± 1.32
0.0	± 2.96	0.0	± 2.59	0.0	± 1.47
0.0	± 3.40	0.0	± 3.02	0.0	± 1.94
0.0	± 4.20	0.0	± 3.71	0.0	± 2.47
0.0	± 4.66	0.0	± 3.92	0.0	± 5.00
0.0	± 4.76	0.0	± 3.94	0.0	± 5.90
0.0	± 8.54	0.0	± 8.13	0.0	± 8.30
0.0	± 9.25	0.0	± 8.84	0.0	± 9.00
0.0	±10.30	0.0	± 9.90	0.0	±10.00
0.0	±12.90	0.0	±12.40	0.0	±13.10

Table 10 Closed-loop eigenvalues for ACOSS-FOUR

Design A		Design B		Design C	
Real part	Imaginary part	Real part	Imaginary part	Real part	Imaginary part
−0.073	± 1.34	−0.050	± 0.92	−0.270	± 0.859
−0.109	± 1.66	−0.073	± 1.23	−0.286	± 1.040
−0.213	± 2.88	−0.216	± 2.38	−0.352	± 1.30
−0.237	± 2.95	−0.242	± 2.58	−0.375	± 1.45
−0.285	± 3.39	−0.291	± 3.01	−0.302	± 1.92
−0.363	± 4.19	−0.366	± 3.69	−0.337	± 2.45
−0.355	± 4.65	−0.363	± 3.91	−0.301	± 5.00
−0.344	± 4.74	−0.359	± 3.93	−0.278	± 5.89
−0.292	± 8.53	−0.291	± 8.12	−0.244	± 8.30
−0.276	± 9.25	−0.274	± 8.84	−0.245	± 9.00
−0.214	±10.30	−0.210	± 9.90	−0.197	±10.00
−0.083	±12.90	−0.081	±12.40	−0.074	±13.10

Table 11 Modal damping parameters of closed-loop eigenvalues for ACOSS-FOUR

Mode	Design A	Design B	Design C
1	0.05464	0.05464	0.3000
2	0.06537	0.05903	0.2660
3	0.07375	0.09042	0.2604
4	0.08016	0.09323	0.2506
5	0.08391	0.09621	0.1551
6	0.08640	0.09866	0.1363
7	0.07608	0.09253	0.0602
8	0.07234	0.09102	0.0471
9	0.03416	0.03579	0.0294
10	0.02982	0.03094	0.0271
11	0.02078	0.02121	0.0196
12	0.00642	0.00651	0.0057

Table 12 Performance index (PI) for different designs

Design	PI
A	739.9
B	721.1
C	781.1

Table 13 Initial cost functions J and E_0 and effective damping response time T_e

J	764.0	732.4	797.6
E_0	2.0	2.0	2.0
T_e	382.0	366.2	398.8

Table 13 contains the effective damping parameter T_e and the magnitudes of J and E_0 defined in Eq. (21). Comparing the plots in Fig. 3, it is seen that the amplitudes of LOS for design C have damped out faster than those for design A or B. The damping characteristics for designs A and B are nearly the same, even though the weight of design B is lower than that of design A. The initial maximum amplitude for design A is equal to 0.29 as against 0.21 for design B. The initial maximum amplitude for design C is equal to 0.24.

Summary

The algorithm used to design a minimum-weight structure with constraints on the modal damping parameters associated with the closed-loop system is presented. Two optimum designs were obtained by using the nominal design of ACOSS-FOUR as the initial design in the iterative optimization algorithm. The equality constraint was imposed on the modal damping parameter and the inequality constraints on the cross-sectional area of the members. In one design, the modal damping parameter was specified to have the same value as that of the nominal design and for the other it was increased by nearly a factor of six over that of the nominal design. The distribution of structural frequencies was different for the optimized structures. The optimum design satisfied the equality constraints exactly. In this investigation, the optimization procedure invokes equality constraints on the closed-loop modal damping ratios without specifying the modal frequencies. In a recent investigation by the authors,[14] the number of equality constraints were increased to include the modal frequencies in order to exclude the significant variation of the fundamental frequency during the optimization process. Also, the optimized designs showed considerable improvement over the nominal design in their dynamic response.

References

[1]Pinson, L. D., Amos, A. K., and Venkayya, V. B. (eds), *Modeling, Analysis and Optimization Issues for Large Space Structures*, Proceedings of NASA/AFOSR Workshop, May 1982.

[2]Khot, N. S., Venkayya, V. B., and Eastep, F. E., "Structural Modifications to Reduce the LOS-Error in Large Space Structures," AIAA Paper 84-0997, May 1984.

[3]Venkayya, V. B. and Tischler, V. A., "Frequency Control and the Effect on the Dynamic Response of Flexible Structures," AIAA Paper 84-1004, May 1984.

[4]Khot, N. S., Venkayya, V. B., and Eastep, F. E., "Structural Modification of Large Flexible Structures to Improve Controllability," AIAA Paper 84-1906, Aug. 1984.

[5]Hale, A. L. and Lisowski, R. J., "Optimal Simultaneous Structural and Control Design of Maneuvering Flexible Spacecraft," *Proceeding of Fourth VPI & SU/AIAA Symposium*, Virginia Polytechnic Institute & State University, Blacksburg, June 1983, pp. 1-16.

[6]Messac, A., Turner, J., and Soosar, K., "An Integrated Control and Minimum Mass Structural Optimization Algorithm for Large Space Structures," Paper presented at Jet Propulsion Workshop on Identification and Control of Flexible Space Structures, San Diego, June 1984.

[7]Salama, M., Hamidi, M., and Demsetz, L., "Optimization of Controlled Structures," Paper presented at Jet Propulsion Workshop on Identification and Control of Flexible Space Structures, San Diego, June 1984.

[8]Junkins, J. L., Bodden, D. S., and Kamat, M. P., "An Eigenvalue Optimization Approach for Feedback Control of Flexible Structures," Paper presented at SECTAM XII Conference, Pine Mountain, GA, May 1984.

[9]Junkins, J. L., Bodden, D. S., and Turner, J. D., "A Unified Approach to Structure and Control System Design Iterations," Paper presented at Fourth International Conference on Applied Numerical Modeling, Taipei, Taiwan, Dec. 1984.

[10]Strunce, R. R., Hegg, D. R., Lin, J. G., and Henderson, T. C., "ACOSS FOUR (Active Control of Space Structures) Theory Appendix," RADC-TR-80-78, Vol. II, 1980, pp. 52-54.

[11]D'Azzo, J. J. and Houpis, *Linear Control System Analysis and Design*, 2nd ed., McGraw-Hill Book Co., New York, 1981, pp. 93-99.

[12]Crane, R. L., "Solution of the General Nonlinear Programming Problem with Subroutine VMCON," Argonne National Laboratory, Argonne, IL, Rept. ANL-80-64, 1980.

[13]Powell, M. J. D., "Algorithms for Nonlinear Constraints That Use Lagrangian Functions," *Mathematical Programming*, Vol. 14, 1978, pp. 224-248.

[14]Khot, N. S., Eastep, F. E., and Venkayya, V. B., "Simultaneous Optimal Structural/Control Modifications to Enhance the Vibration Control of a Large Flexible Structure," *Proceedings of the AIAA Guidance, Control and Navigation Conference*, Snowmass, CO, Aug. 1985, pp. 459-466.